DE BROGLIE RELATION AND HEISENBERG UNCERTAINTY PRINCIPLE AND ITS SIGNIFICANCE

What is De Broglie's equation?

The de Broglie equation included several issues in its findings, such as whether only light exhibits dual nature or there are other particles as well? How can the research be concluded in this matter, what objects should be taken into consideration, etc. de Broglie proposed that particles had a wave-like character. Radiation may act as both waves and particles. Different things can get energy and momentum from both radiation and moving particles. Quantum theory was founded on the dual nature of both radiation and matter.

De Broglie postulated that, similar to how light exhibits wave-like and particle-like characteristics, matter also possesses wave-like and particle-like features. The dual behaviour of matter was described as its fundamental characteristic. De Broglie derived a correlation between wavelength and momentum of matter.

The De Broglie equation depicts the relationship between a particle's mass, velocity and wavelength. According to the De Broglie equation, the wavelength is proportional to the particle's mass and velocity (h is Planck's constant, 6.626×10-34 J.s). This helps explain why this wavelength is so small that large objects cannot see it. However, the particle's wavelength is minuscule like that of an electron's (m = 9.11×10-31kg) as predicted by the De Broglie equation.

What is the significance of the De Broglie relationship?

A De Broglie wave, also known as a matter wave, is any feature of a material object's behaviour or qualities that fluctuates in time or space according to the mathematical equations that describe waves. However, the wave character of matter has no bearing on everyday objects of usual sizes since the wavelength of the wave associated with them is too small to be recognised.

Deriving the De Broglie Relationship:

The behaviour of a very low-mass particle travelling at speed less than that of light is similar to that of a particle and a wave. De Broglie devised a formula that relates the mass of smaller particles to their wavelength.

Given the nature of particles, Einstein's equation is as follows:

$E = mc^2$ …………… (1)

Where,

E = energy

m = mass

c = speed of light

Considering the wave nature, Planck's equation is given as,

$E = h\, v$ ………….. (2)

Where,

E = energy

h = Planck's constant

v = frequency

From (1) and (2),

$mc^2 = h\, v$……………(3)

Frequency, (v) can also be expressed as λ (in terms of wavelength) as,

$v = c/\lambda$

Now, c can be replaced with the object's velocity, v, about the general particles. Therefore, equation (3) can be written as,

$m\, v^2 = h\, v/\lambda$

I.e., $\lambda = h/m$

The above equation is known as the De Broglie relationship, and the wavelength λ is known as the De Broglie wavelength.

The Heisenberg Uncertainty Principle

The Heisenberg uncertainty principle is a significant principle in quantum mechanics. The dual nature of a wave-particle is the major reason for the genesis of the uncertainty principle.It defines the limitations to which the position and momentum of a particle may be known simultaneously. It argues that observing an electron with a microscope necessitates a photon reflecting the electron. The reflected photon alters the electron's route.

For instance, the interaction of an electron with photons of light can be used to detect electrons. However, since photons and electrons have roughly the same energy, using a photon to locate an electron will knock the electron off track, leaving the electron's location unknown.

The wave-particle duality of matter underpins this principle. Although Heisenberg's uncertainty principle may be ignored in the macroscopic world, it is imperative in the quantum world.

Because we cannot quantify accurate energy in a specified period of time, the same connection may be drawn between time and energy. It can be concluded that Heisenberg defined uncertainty in different pairs of time and energy and position and momentum, as having a minimum value equal to the Planck's constant divided by 4π.

The theory of Heisenberg's uncertainty principle

German physicist Werner Heisenberg introduced the uncertainty principle in 1927. According to Heisenberg's uncertainty principle, it is impossible to precisely identify both the position and velocity of particles that exhibit both particles and wave properties simultaneously.

According to the uncertainty principle, we cannot precisely measure a particle's location (x) or momentum (p). The more we know about one of these qualities, the less accurately we know about the other. Planck's constant is a crucial quantity in quantum theory since it allows us to quantify the world's granularity, and it has the value of 6.626 x 10-34 joule seconds.

If Δx is the error in position measurement and Δp is the error in the measurement of momentum, then

$X \cdot p > h/4\pi$

As, $p = mv$, thus the uncertainty principle formula can be expressed as-

$X \cdot mv \geq h/4\pi$ or

$X \cdot m \times \Delta v \geq h/4\pi$

Assume that mass remains constant and that ∆V is the measurement error in velocity, then the expression becomes –

$X \cdot V \geq h/4\pi\ m$

Now, When the Heisenberg principle is applied to an electron in an atom's orbit, with $h = 6.626 \times 10^{-34}$Js and $m= 9.11 \times 10^{-31}$Kg, then

$X \cdot V \geq 6.626 \times 10^{-34}/ 4 \times 3.14 \times 9.11 \times 10^{-31} = 10^{-4} m^2 s^{-1}$

If the electron's position is adequately measured with accuracy to its size (10^{-10}m), then the error in velocity measurement will be equivalent to or greater than 1000km or 10^6m.

We generally consider two standard equations related to the uncertainty principle in practice. They are;

$X \cdot p > h/4\pi$

$E \cdot t > h/4\pi$

Where,

h = value of the Planck's constant or (about 6.6×10^{-34} joule-second)

t = uncertainty in time measurement

X = uncertainty in the position

E = uncertainty in the energy

p = uncertainty in momentum

Significance / Limitation of Heisenberg uncertainty principle

The de Broglie equation depicts the relationship between a particle's mass and velocity and its dependence on wavelength. The Heisenberg uncertainty principle quantifies the limitations to which the position and momentum of a particle can be known at the same time. The product of position and momentum uncertainty will always be larger than h/4**π**. This indicates that the more precisely the position is known, the more uncertain the momentum is, and vice versa.

The uncertainty principle is relevant only in the context of microscopic particles, since the energy of photons is not capable enough to shift the position and velocity of larger bodies. The limitation of the principle is that it applies only to dual-natured microscopic particles. Any particle whose wave nature is very small shall not depict Heisenberg's uncertainty principle.